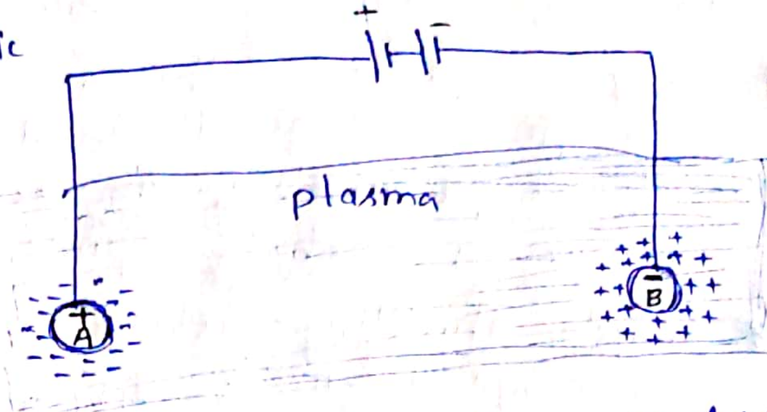
Debye's sphere, Debye's shielding distance (or Debye's screening distance or Debye's length or Debye's shielding radius) & Debye's potential :-



A fundamental characteristic of plasma is its ability to shield out electric potential that are applied to it.

When we insert a source of charge or probe charge into the body of plasma then we see that the effect of that probe charge is shielded in a finite volume of the plasma and rest part of the plasma remains unaffected from that probe charge.

In shown figure, two balls A and B are connected with a battery so that the ball A gets positively charged and the ball B gets negatively charged. If we consider +Q charge on the ball A then −Q charge will be on the ball B.

Since plasma contains positively charged ions and negatively charged electrons so positively charged ball A attracts negatively charged electrons due to electrostatic attractive force and the ball A is surrounded by electrons. Similarly the ball B attracts positively charged ions and the ball B is surrounded by positively charged ions. If we consider plasma is cold and there is no thermal motion of the charged particles then the charge surrounded the sphere is equal to probe charge (charge of sphere) in magnitude but opposite in nature (sign). That means total charge of the imaginary sphere (probe charge plus charge surrounded the sphere) is zero so the effect of this charge is not felt by outside the imaginary sphere.

This imaginary sphere in which the effect of the probe charge (charge of ball) is shielded is known as Debye's sphere. The distance upto which the effect of the probe charge is shielded, is known as Debye's shielding

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distance or Debye's screening distance or Debye's length or Debye's shielding radius. Radius of Debye's sphere is Debye's shielding distance.

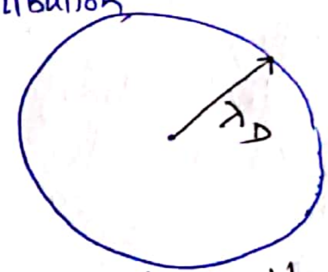
* Debye's sphere :- The imaginary sphere surrounded a probe charge in a plasma in which the effect of the probe charge in the plasma is shielded, is known as Debye's sphere.

* Debye's shielding distance or Debye's screening distance or Debye's length or Debye's shielding radius :- The radius of Debye's sphere is known as Debye's shielding distance. It is denoted by λ_D.

* Debye's potential :- The potential on the surface of Debye's sphere, is known as Debye's potential.

Expression for Debye's potential and Debye's shielding distance or Debye's length (λ_D) :-

As there is continuous charge distribution so we may apply poisson equation



Debye's sphere.

$$\nabla^2 \phi = -\frac{\rho}{\varepsilon_0} \quad \text{——— (1)}$$

where $\nabla^2 \rightarrow$ Laplacian operator
$\phi \rightarrow$ Electric potential
$\rho \rightarrow$ volume charge density
$\varepsilon_0 \rightarrow$ permittivity of free space.

Let n_i = number density of positive ions
n_e = number density of free electrons
$+e$ = charge on one ion
$-e$ = charge on one electron.

So volume charge density $\rho = n_i e + (-n_e \cdot e)$

$\Rightarrow \rho = (n_i - n_e) \cdot e$ put in equ^n (1), we get

$$\nabla^2 \phi = -\frac{(n_i - n_e)e}{\varepsilon_0} \quad \text{——— (2)}$$

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As we know that electrons and ions follow the Maxwellian Boltzmann's Distribution in plasma so

From Maxwellian-Boltzmann's Distribution

$$n_i = n_0 \cdot e^{-e\phi_i/KT} \quad \text{and} \quad n_e = n_0 e^{e\phi_e/KT}$$

Where n_0 = Average number density of charged particles.

ϕ_i = potential due to ions

ϕ_e = potential due to electrons

K = Boltzmann's constant

T = Absolute temperature.

Now $n_i - n_e = n_0 \left[e^{-e\phi_i/KT} - e^{e\phi_e/KT} \right]$

If $e\phi << KT$ or $\frac{e\phi}{KT} << 1$ then in this case, terms of higher power of $\frac{e\phi}{KT}$ can be neglected.

$e^x = 1 + x + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \dots$ but $e^x = 1 + x$ for $x << 1$

Using these, we get

$$n_i - n_e = n_0 \left[1 - \frac{e\phi_i}{KT} - 1 - \frac{e\phi_e}{KT} \right]$$

$$\Rightarrow n_i - n_e = -\frac{n_0 e}{KT} (\phi_i + \phi_e) \quad \text{——— (3)}$$

Using equ (3) in equ (2) we get

$$\nabla^2 \phi = \frac{n_0 e^2}{\varepsilon_0 KT} (\phi_i + \phi_e) \quad \text{——— (4)}$$

Mobility of ions is very small in comparision with mobility of electrons because ion is much heavier than electron. So electron is main source of current contribution in plasma and this is why ions only serve to keep the plasma neutral. So potential due to ions is much less than potential due to electrons.

Since $\phi_i << \phi_e$ so $\phi_i + \phi_e \approx \phi_e = \phi$ (let)
put in equ (4).

$$\nabla^2 \phi = \frac{n_0 e^2}{\varepsilon_0 KT} \cdot \phi$$

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or $\nabla^2\phi = K_D^2\,\phi$ ——— (5)

where $K_D = \left(\frac{n_0 e^2}{\epsilon_0 K T}\right)^{1/2}$ ——— (6)

If $\phi = \phi(x, y, z)$ then from eqn (5)

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = K_D^2\,\phi \rightarrow \text{Three dimensional.}$$

If $\phi = \phi(x)$ only one dimensional then from eqn (5)

$$\frac{\partial^2\phi}{\partial x^2} = K_D^2\,\phi$$

or $$\frac{\partial^2\phi}{\partial x^2} - K_D^2\,\phi = 0 \text{ ——— (7)}$$

Solution of eqn (7) is given by

$$\phi(x) = A\cdot e^{K_D x} + B\cdot e^{-K_D x} \text{ ——— (8)}$$

Where A and B are arbitrary constants.

A and B can be obtained by certain boundary condition.

(i) $\phi = 0$ when $x \to \infty$ (ii) $\phi = \phi_0$ when $x = 0$

Using condition (i) in eqn (8), we get

$$0 = A\cdot e^{\infty} + B\cdot e^{-\infty} = A\cdot e^{\infty} + B\cdot 0 \quad \because e^{-\infty} = 0$$

$\Rightarrow A\cdot e^{\infty} = 0 \Rightarrow \boxed{A = 0}$ because $e^{\infty} \neq 0$

So $\phi(x) = B\cdot e^{-K_D x}$ ——— (9)

Now using condition (ii) in eqn (9), we get

$\phi_0 = B\cdot e^{0} \Rightarrow B = \phi_0$

Hence $\phi = \phi_0\, e^{-K_D x}$

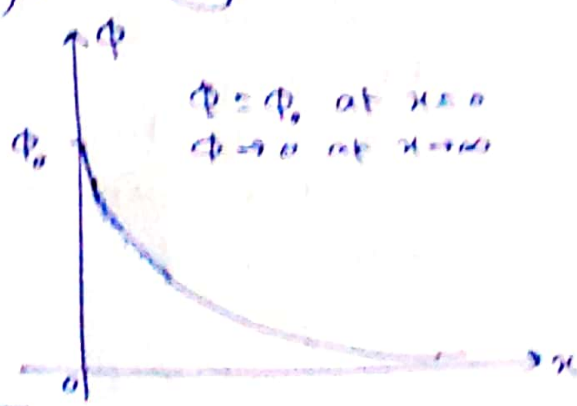
or $\boxed{\phi = \phi_0\cdot e^{-x/\lambda_D}}$ ——— (A)

where $\lambda_D = \frac{1}{K_D} \Rightarrow \lambda_D = \left(\frac{\epsilon_0 K T}{n_0 e^2}\right)^{1/2}$ ——— (B)

ϕ = Debye's potential.

λ_D = Debye's length or Debye's shielding distance or Debye's screening distance.

$\lambda_D \propto T^{1/2}$
$\quad\propto n_0^{-1/2}$ $\Big\}$ If $n_0 \to \infty$ then $\lambda_D = 0$



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